

Pls state all the theorems that you are using irrespective of how simple it might be

There is no time limit for this paper.

You might sit as much as you want to and write. Only condition you cannot go out in between.

Best of luck !!!

Full marks that can be earned is 200

- Let  $f(x) = \frac{1}{1+25x^2}$  in the interval  $[-1, 1]$ . Consider approximation of this function at equidistant interpolation points by both fourth and fifth degree polynomial. Compare and comment on the approximation for  $|x| \geq 0.726$  and  $|x| \leq 0.5$

40 marks

- Prove that there exists a unique trigonometric polynomial

$$q_n(t) = \frac{a_0}{2} + \sum_{i=1}^n [a_i \cos(it) + b_i \sin(it)]$$

satisfying the interpolation property

$$q_n\left(\frac{2\pi j}{2n+1}\right) = y_j, 0 \leq j \leq 2n$$

Coefficients are given by

$$a_i = \frac{2}{2n+1} \sum_{j=0}^{2n} y_j \cos\left(\frac{2\pi j i}{2n+1}\right), 0 \leq i \leq n$$

$$b_i = \frac{2}{2n+1} \sum_{j=0}^{2n} y_j \sin\left(\frac{2\pi j i}{2n+1}\right), 1 \leq i \leq n$$

40 marks

- Prove that if  $f \in C[0, 1]$  vanishes at 0 and 1 then the sequence  $(B_n^* f)$

$$B_n^*(f, x) = \sum_{k=1}^{n-1} \left[ \binom{n-1}{k} f\left(\frac{k}{n}\right) \right] x^k (1-x)^{(n-k)}$$
 converges uniformly to  $f$

50 marks

- Do you think B-splines  $B_m$  on the common interval  $\left[\frac{(m-1)}{2}, \frac{(m+1)}{2}\right]$  are linearly independent. If so give a proof else provide a counter example

20 marks

- State and prove Korovkins theorem

20 marks

- Write down the algorithm for the Fast Fourier Transform. Analyze the time complexity of the algorithm.

30 marks